Optimal Dispatch of Active and Reactive Generation using Quadratic Programming

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Outline

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Motivation

- Maintaining continues balance between generation and load demand
- Maintaining system frequency, voltage level and security at minimum cost
- Minimizing transmission losses
- Even distribution of spare capacity
- Various methods are used to achieve these goals, one of which is by the use of quadratic programming technique

Quadratic programming

- General quadratic programming problem can be expressed in the form,

$$\text{Max } f(X) = CX + X^T DX + \text{constant} \qquad \text{(n variables)}$$

$$\text{S t } AX \leq b \qquad \text{(m constraints)}$$

$$X \geq 0 \qquad \text{(n variable constraints)}$$

Where,$X = [X_1, X_2....X_n]^T$ is a nx1 vector consisting set of decision variable

D- nxn dimensional real symmetric matrix

C is a real valued n –dimension vector

- The objective function has two part
  - Quadratic part - $X^T DX$
  - Linear part – CX and the constraints

Quadratic programming continued..

- General "Kuhn–Tucker conditions" of nonlinear programming for solution to be optimal,

$$\lambda \geq 0$$

$$\nabla f(X) - \lambda \nabla g(X) = 0$$

$$\lambda_i g_i(X) = 0$$

$$g(X) \leq 0$$

Applying Kuhn–Tucker condition to quadratic programming problem,

$$\lambda , \mu \geq 0$$

$$C + 2X^T D - (\lambda \;\; \mu) (A \; -I)^T$$

$$\lambda_i g_i(X) = 0$$

$$g(X) = (A \;\; -I)^T X - (b \; 0)^T \leq 0$$

$\mu = [\mu_1, \mu_2 ..... \mu_n]$ Lagrangian multiplier (as many as number of variables = n)

$\lambda = [\lambda_1, \lambda_2 .... \lambda_m]$ Lagrangian multiplier (as many as number of constraints = m)

Quadratic programming continued..

➢Previous equation can be re-written as

$$\lambda , \mu \geq 0$$

$$C = -2X^{T}D + (\lambda \;\; \mu) (A -I)^{T} = 0$$

$$\lambda_i \; (A_i X_i - b) = 0$$

$$\mu_j X_j = 0$$

$$AX \leq b$$

$$X \geq 0$$

➢Since $AX \leq b$, we can introduce a slack variable "S" such that

$$AX + S = b$$

$$\lambda_i S_i = 0$$

Quadratic programming continued..

- So now the Kuhn–Tucker conditions reduced to following sets of equation

$$-2X^TD + \lambda^T A - \mu^T = C \quad \text{.................................. (1)}$$

$$AX + s = b \quad \text{..................................(2)}$$

$$\lambda_i S_i = 0 \quad \text{..................................(3)}$$

$$\mu_j X_j = 0 \quad \text{..................................(4)}$$

$$\lambda \; \mu \geq 0 \quad \text{..................................(5)}$$

$$X \geq 0 \;\; s \geq 0 \quad \text{..................................(6)}$$

Quadratic programming continued..

$$\begin{bmatrix} -2D & A & -I & 0 \\ A & 0 & 0 & I \end{bmatrix} \begin{vmatrix} X \\ \lambda \\ \mu \\ s \end{vmatrix} = \begin{vmatrix} c \\ b \end{vmatrix}$$

$$X\ \lambda\ \mu\ s \geq 0$$

- Solving for X, λ, μ,s such that it satisfies all these equation, give us the best outcome possible for a given problem
- Hence this method is applied in minimizing generation cost in power scheduling

Formulation of the problem

- Production cost problem is modeled as quadratic programming problem with system operating constraints
- Provides optimum solution for real power dispatch in finite number of exchange steps which avoids complex nesting of iterative loops
- Optimum allocation of reactive power using gradient technique

Modeling the problem

- Generator dispatch problem concerned with scheduling a generator output "X", based on predicted load so that system cost is minimized
- So generation cost is considered in the form

$$F(X) = a + d^T X + X^T W X \quad \text{.......................} \ (1)$$

Where, X – $[X_1, X_2....X_n]^T$ Vector of generator active power output

a – generator fixed running cost

d – m-vector of generation linear cost co-efficient

W – m-diagonal matrix of the generator quadratic cost coefficient

Plant constraints

- Permissible loading constraints on generator (X), station (S) and group outputs are specified by upper and lower power limits

$$X^L \le X \le X^U \qquad S^L \le S \le S^U \qquad S_j = \Sigma_{i \in j} X_i \; i=1,2,..m \qquad \text{......................} (2)$$

- Limit on rate of change of real power output

$$\nabla X^L \le \nabla X \le \nabla X^U \qquad \nabla S^L \le \nabla S \le \nabla S^U \qquad \text{......................} (3)$$

- Spare plant capacity must be available on demand to supply unscheduled generation in case of error

$$\Sigma_{S=1 \text{ to } P} K_s X_s + R_0 > R \qquad \text{......................} (4)$$

Where, R_0 maximum spare capacity available, R is a spare requirement

Ks is a coefficient, X, S Vector of generator and station active power output respectively

Network constraints

- For power balance of total generation

$$P_N + P_L(X) = \Sigma_{i=1 \text{ to } m} X_i \quad \text{.......................} (5)$$

Where, P_N – Network load, P_L – Transmission loss

- Restriction on nodal reactive power and voltage are considered with

$$Q^L \leq Q_g \leq Q^U \qquad V^L \leq V \leq V^U \quad \text{...................} (6,7)$$

- Transmission line power flow must also be limited to prevent overloading with constraint related to maximum MVA rating
- Generator reactive power output are assumed to be variable and independent of real power output

Generalizing the constraint

- Constraints involving real power limitations (equation 2-5) are incorporated in to quadratic programming structure
- In general all real power constraint are approximated by linear combination of generator output of the form

$$A\,X \le E$$

Where, A – qxm matrix of constant coefficient , E – q-vector of constraint

- Constraints involving reactive power limitations are satisfied by gradient allocation technique
- In addition we can include transmission loss

Real power dispatch

- Total transmission power loss (P_L) is related to nodal currents I and symmetrical bus impedance matrix Z_N

$$P_L + j\,Q_L = I^{*T} Z_N\, I \qquad \ldots\ldots\ldots\ldots\ldots\ (8)$$

Where, $I = I_r + j\,I_i$ (vectors of nodal current with real and reactive power)

$$P_L = I_r^T R\, I_r + I_i^T R\, I_i = \Sigma_{j=1 \text{ to } n} \Sigma_{k=1 \text{ to } n} (I_{kr}\, r_{kj}\, I_{jr} + I_{ki}\, r_{kj}\, I_{ji}) \qquad \ldots\ldots\ldots\ldots\ldots\ (9)$$

- Also with net power $P_k + jQ_k$ at bus K

$$I_k^* = (P_k + jQ_k) / \{ |V_k| (\cos\theta_k + j\sin\theta_k) = I_{kr} - j\,I_{ki} \qquad \ldots\ldots\ldots\ldots\ (10)$$

Where,

$$I_{kr} = (P_k \cos\theta_k + Q_k \sin\theta_k) / |V_k|$$

$$I_{ki} = (P_k \cos\theta_k - Q_k \sin\theta_k) / |V_k| \qquad \ldots\ldots\ldots\ldots\ (11)$$

Real power dispatch continued..

- Now equation (9) can be written as,

$$P_L = \Sigma_{j=1 \text{ to } n} \Sigma_{k=1 \text{ to } n} (P_k \alpha_{kj} P_j + P_k \beta_{kj} Q_j - Q_k \beta_{kj} P_j + Q_k \alpha_{kj} Q_j) \text{ .... } (12)$$

Where,

$$\alpha_{kj} = r_{kj} \cos \theta_{kj} / (|V_k| |V_j|)$$

$$\beta_{kj} = - r_{kj} \sin \theta_{kj} / (|V_k| |V_j|) \text{ ......}(13)$$

- Same can be written in a matrix form with

$$\alpha = [\alpha_{kj}] = \alpha^T \text{ and } \beta = [\beta_{kj}] = - \beta^T , \boldsymbol{\pi}/2 \geq \theta \geq - \boldsymbol{\pi}/2$$

- So the real power related to network parameters and steady state data

$$P_L = P^T \alpha P + 2 P^T \beta Q + Q^T \alpha Q$$

Real power dispatch continued..

- In terms of load and generator power component at each node with

$$P_k = P_{kl} + P_{kG,}\ k = 1, 2 ,..\ n$$

$$P_L = P_G^T \alpha\, P_G + 2\,(P_l^T \alpha - P_l^T \beta)\, P_G + (P_l^T \alpha - 2\, Q^T \beta)\, P_l + Q^T \alpha\, Q \qquad \text{.......} (15)$$

- With m generator connected to n-nodes,

$$P_L = X^T K\, \alpha\, K^T X + 2\,(P_l^T \alpha - Q^T \beta)\, K^T X + (P_l^T \alpha - 2\, Q^T \beta)\, P_l + Q^T \alpha\, Q \text{.......} (16)$$

Where,

k = mxn generator network node connection matrix with elements Kij (of Kt) = (1,0) if generator j is incident or not with node i

Real power dispatch continued..

- For real power optimization, the cost function is given by

$$\Psi(x) = F(X) + \mu X^T \alpha K K^T X + 2 \mu (P_l^T \alpha - Q^T \beta) K^T X \quad ....(17)$$

Where μ – average cost of received power (=Total production cost/ ΣX)

- Including equation (1),

$$\Psi(x) = c_\infty + 2 C_o X + X^T C X \quad ....(18)$$

Where $2 C_o - d^T + 2 \mu (P_l^T \alpha - Q^T \beta) K$

$C - \mu X^T \alpha K^T + W$, which is symmetric and Ψ(x) is positive definite..

Reactive Power Dispatch

- It is based on minimization of transmission loss function by variation of source reactive output power Q_g
- Magnitude constraint is of the form

$$Q^L \leq Q_g \leq Q^U$$

- Steepest descent approach allocates the reactive power such that at iteration k, the new allocation is given by

$$Q_{(k+1)} = Q_k - h(\Delta P_L)_K \quad \text{..........................(19)}$$

Where, $(\nabla P_L)_K$ is a n-gradient vector

$$(\nabla P_L)_K = (\partial P_L/\partial Q)_k = 2(\alpha\, Q - \beta\, P)_k \quad \text{..........................(20)}$$

h – step length to be taken in the direction given by $(\Delta P_L)_K$

Reactive Power Dispatch continued..

- At any iteration K, overall change in the reactive power will be small compared with the total
- Average reactive generation change at each iteration should be approximately zero

$$\delta = 1/r\ (\Sigma_{i=1 \text{ to } n\ (i \in j)} \nabla P_{l_i} \qquad \text{.................................(21)}$$

Where j – Node to which reactive sources are connected

r – number of such nodes

New bus power determined by gradient step length

$$Q_{(k+1)} = Q_k - h\ \acute{k}\ \{\ (\nabla P_L)_k - \delta\ I_n \qquad \text{.................................(22)}$$

Where I_n – n dimensional unit vector with node current

$\acute{k}$ - nxn matrix indicating which nodes have reactive sources

Reactive Power Dispatch continued..

- Change in power loss with alteration in the reactive bus power

$$\Delta P_{L} = \Delta Q^{T} (\Delta P_{L})_{K} = h (\Delta P_{L}^{T})_{k} (\Delta P_{L})_{k} = (P_{L})_{k} - (P_{L})_{k+1} \qquad \text{.....................(23)}$$

- Since Real power distribution $(P)_{K}$ and coefficients $(\alpha)_{K}$ and $(\beta)_{k}$ remain constant for the reactive dispatch, real power loss reduces to

$$\Delta P_{L} = (P^{T})_{k} (\beta)_{k} - \{ Q_{k} - Q_{k+1} \} + (QT \nabla P_{L} /2)_{k} \qquad \text{.....................(24)}$$

- Convergence is achieved by calculating "h" using true gradient

$$h = \{ Q^{T} \Delta P_{L} / 2 \Delta P_{L}^{T} (\Delta P_{L} + \beta P)\}_{k} \qquad \text{...................(25)}$$

- If voltage constraints are violated in an a.c load flow based on the new reactive power dispatch, the calculation of $Q_{(k+1)}$ are repeated with the reduced step length..

Load Forecasting

- Accurate forecasting model is required for economic generator scheduling
- Spectral expansion method is used for predication
- Prediction is completely based on past load data
- This avoids possible inaccuracy introduced because of inaccurate weather forecast

Optimization Procedure

- Scheduling problem requires minimization of convex quadratic function of m-variable

$$\Psi(x) = c_{\infty} + 2\, C_o X + X^t C X$$

- Subject to Q linear inequality constraint

$$AX \leq E\ ,\ X \geq 0$$

- Inequality constraint transformed in to equation with slack variable y

$$(A,\ I) \begin{pmatrix} X \\ y \end{pmatrix} = E = \bar{A}\,\bar{X}$$

$$\text{and} \quad y \geqslant 0$$

Where Ā is of the order q x (q+m)

Optimization Procedure continued..

- The problem of finding a basic feasible solution is same in linear and quadratic programming
- The constraints may then be used to express basic variables "Y"

$$Y = Y_1, Y_2.....Y_q$$

In terms of currently non basic variables "Z"

$$Z = Z_1,Z_2.....Z_m$$

As, $$Y = (A_0 + AZ)_1$$

Where index 1 indicates value at the initial trail solution

A – Constraints co-efficient matrix

Optimization Procedure continued..

➢Equating these variables in objective function in equation (18)

$$\Psi(x) = c_{\infty} + 2\, C_o X + X^t C X$$

We get,

$$\Psi(x) = (c_{\infty} + 2\, C_o Z + Z^t C Z)_1$$

➢Rearrangement of the equation gives,

$$\Psi(Z)_1 = (\hat{Z}^T C \; \hat{Z})_1$$

Where, $(\hat{Z}) = (1, Z_1, Z_2 \ldots\ldots Z_m)_1^T$ and

$$(C)_1 = \begin{bmatrix} C_{\infty} & C_o \\ C_o^t & C_1 \end{bmatrix}_1$$

Of the order (m+1)x(m+1)

Optimization Procedure continued..

- If $(C_1)_1$ is symmetric and the quadratic form $X^T (C_1)_1 X$ is semidefinite, then for the kth step

$$(\partial\psi/\partial Z)_k = 2(C_o^t + C_1 Z)_k = 2(C\bar{Z})_k$$

- The Kuhn–Tucker conditions for optimality are satisfied if $(\partial\Psi/ \partial Z)_k \geq 0$ and trial point chosen is the optimal solution
- If optimum has not reached, then for certain Z_i, $(\partial\Psi/ \partial Z)_k < 0$ holds at the trial point and Ψ may be reduced by making Z_i positive

Optimization Procedure summary

1. Input transmission system data, operating limit and initial conditions
2. Calculate bus impedance matrix Zn
3. Establish network loading and perform ac load flow using Gauss-Seidel iteration technique
4. Calculate transmission losses
5. Calculate total generation required (load + losses)
6. Solve quadratic problem for real power dispatch

Optimization summary continued..

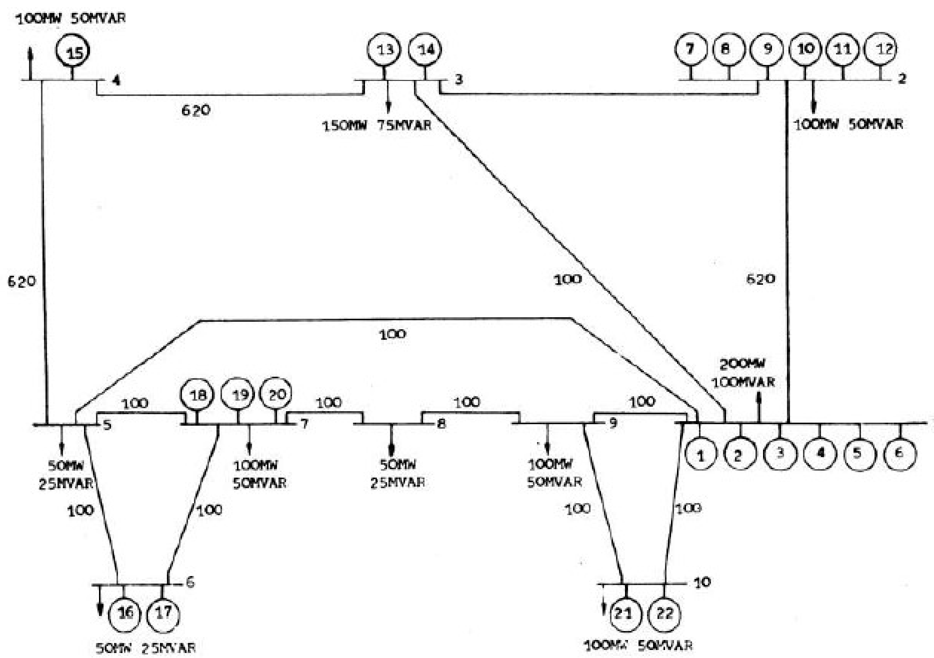
7. Perform ac load flow with new real power dispatch
8. Calculate transmission losses
9. Test if losses have converged, if not go to step 5
10. Determine system production cost
11. Is reactive power dispatch is required? If not, go to step 15
12. If previous reactive dispatch has been made, has the system production cost converged? If so, go to step 15

Optimization summary continued..

13. Allocated reactive power using gradient method
14. Calculate transmission losses and go to step 5
15. Output load flow results, production coast and appropriate schedules

Testing

- ➢This procedure was applied to 22-generator and 10-node section of the British grid system



- ➢Comparison of Production cost, Real power loss and number of iterations has been made with linear approximation technique

Results - Generator dispatch

Gen.No.	Lower MW limit	Upper MW limit	Linear cost d £/MW.hr	Quadratic cost w £/MW^2.hr	L.P.Dispatch MW	L.P.Dispatch MVAR	Q.P.Dispatch MW	Q.P.Dispatch MVAR
1	10	60	1.9	0.01	60	27.5	30	26.7
2	10	60	1.9	0.01	60	27.5	30	26.7
3	10	60	2.0	0.01	17	27.5	25	26.7
4	10	60	2.0	0.01	10	27.5	25	26.7
5	10	60	1.9	0.015	60	27.5	20	26.7
6	10	60	1.9	0.015	60	27.5	20	26.7
7	20	100	2.0	0.0	100	17.5	100	17.1
8	20	100	2.0	0.0	100	17.5	100	17.1
9	20	100	2.0	0.0	100	17.5	100	17.1
10	20	100	2.0	0.0	100	17.5	100	17.1
11	20	100	2.0	0.0	50*	17.5	50*	17.1
12	20	100	2.0	0.0	50*	17.5	50*	17.1
13	10	60	2.0	0.01	60	5.1	24	4.6
14	10	60	2.0	0.01	10	5.1	24	4.6
15	20	50	2.1	0.0	20	91.3	50	80.8
16	10	60	2.2	0.0	10	14.6	50	16.3
17	10	60	2.2	0.0	10	14.6	50	16.3
18	5	30	1.95	0.02	30	25.2	18	26.2
19	5	30	1.95	0.02	30	25.2	18	26.2
20	5	30	1.95	0.02	30	25.2	18	26.2
21	10	60	2.0	0.005	10	36.6	56	37.2
22	10	60	2.0	0.005	36	36.6	56	37.2
					COST=£2284.9 LOSS=12.8MW		COST=£2135.3 LOSS=10.1MW	

Results – Transmission line power flows

Line code		Impedance p.u. R	X	L.P. power flow MW	MVAR	Q.P. power flow MW	MVAR
1	2	0.003	0.028	−142	13	−173	13
1	3	0.059	0.151	−7	8	−14	9
1	5	0.143	0.364	21	1	9	2
1	9	0.044	0.112	96	32	77	29
1	10	0.029	0.073	99*	12*	49	7
2	3	0.001	0.010	257	63	226	58
3	4	0.004	0.032	170	−1	108	−5
4	5	0.005	0.042	89	32	58	23
5	6	0.055	0.140	32	−1	−20	−4
5	7	0.073	0.185	27	3	37	1
6	7	0.132	0.336	1	2	29	3
7	8	0.029	0.073	18	29	18	28
8	9	0.033	0.084	−33	4	−32	2
9	10	0.033	0.084	−41	−26	−58	−26

Results summary

- Production cost using proposed method was £2135.3/hr compared with £2284.9/hr for linear approximation
- Corresponding real power loss were 10.1 MW and 12.8 MW
- More uniform voltage profile is obtained with quadratic model
- Even distribution of spare capacity
- Solution of the full problem required three real power and two reactive power dispatch for convergence

Conclusion

- Optimization technique can be adopted for continues economic scheduling combined with automatic load frequency control
- Relatively fast solution can be obtained for real and reactive power dispatch
- Since nonlinearities in the generator cost curve can be included in the quadratic formulation, with reduced transmission loss, it produces more economic and uniform power distribution
- This method can be incorporated with continues load prediction with weather forecasting , can be integrated in to an automatic power system control scheme.

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Thank you!